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Research Article

Robust Linear Regression Estimation via Symlet Andrews Weighted Function for Outlier Detection

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ABSTRACT

This paper suggests a novel method for robust estimation in linear regression models by fusing the Andrews weighted function with Symlet wavelet analysis. The result is a new weighted function that includes the Symlet Andrews Weighted Function. These functions try to reduce the effect of outliers in regression analysis by giving data points with high residuals lower weights. The effectiveness of these new weighted functions was evaluated using simulated experiments, using the root mean square error (RMSE) as the standard. The results show that, in terms of RMSE values, the recommended weighted function consistently performs better than the traditional weighted function. This improvement suggests that because the new function can handle outlier data, they are more reliable for linear regression analysis, including problematic data.

Keywords: Symlet wavelet, Andrews weighted function, robust estimation, outliers, linear regression

INTRODUCTION

Since traditional implementations of regression analysis are infamously sensitive to outliers, robust estimate techniques that preserve accuracy in the face of abnormal data have been developed. Regression analysis is the primary tool for modeling connections between variables. Weighting functions, which give points with high residuals less weight to keep them from skewing the model's fit, are commonly used in these robust techniques. In addition to these statistical methods, the wavelet transformation (WT) offers an advanced time-frequency analysis tool that breaks down data into several scales, managing non-stationary trends and making de-noising easier. Together, these ideas enable sophisticated signal processing and regression modeling, where robust estimation and weighting functions protect the model's integrity from noise and contamination while wavelets handle intricate signal structures. However, if the data do not meet some of these assumptions, sample estimates and results may be misleading. Outliers specifically defy the least squares regression that the residuals are regularly distributed.[1] Integrating efficiency into the weight function is challenging since it must be written as a function in the equation, and the shape of the weight function is directly derived from optimum control in this study. Research focuses on how to depict the rise and fall of the weight function utilizing the qualities of a robust function to boost robustness's role in general. These regression domains are frequently explored because outliers or influencing data have a significant impact on regression conclusions.[2]

RESEARCH OF METHODOLOGY

This work proposes a new approach to robust estimation in linear regression models by combining Symlet wavelet analysis with the Andrews weighted function. The outcome is a new weighted function that incorporates the Symlet Andrews weighted function (SAWF). By assigning lower weights to data points with high residuals, these functions attempt to lessen the impact of outliers in regression analysis. Simulated experiments were used to assess the efficacy of these new weighted functions.

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Multiple Linear Regression Model

Regression analysis is a statistical method used to explain the relationship between response and predictor variables. The models may also be used to determine how important the dependent variable is to the independent variable.[3] The following notation is used to denote a multiple regression model, which frequently includes numerous independent variables:

$$Y_i = \beta 0 + \beta_1 X_{i1} + \beta_2 Xi_2 + \beta_3 X_{i3} + \cdots + \beta_k X_{ik} + \varepsilon_i \quad (1)$$

where,
Y_i is the dependent variable.
X_{i1}, X_{i2}., X_{ik} the independent variable's value on the i-th observation. Regression parameters are $\beta_0, \beta_1, \ldots, \beta_k$.,
ε_i is a normally distributed random variable.

A multiple linear regression model with n observations and k explanatory variables, for instance, might be described as follows:

$$Y = X\beta + \varepsilon \quad (2)$$

Where $Y_{n\times1}$ is a response vector, $X_{n\times(k+1)}$ is a non-stochastic input matrix, $\beta_{(k+1)\times1}$ is an unknown coefficient vector, and $\varepsilon_{(n\times1)}$ is an error vector distributed normally with $E(\varepsilon) = 0$ and $V(\varepsilon) = \sigma^2 I_{n.}$[4]

One of the most popular approaches for estimating the model parameters in equation (3) is the ordinary least squares (OLS) method. This approach minimizes the sum of squares of residuals, which is defined as follows: The least squares method provides the best unbiased linear estimation.

$$SSR = (Y - X\beta)^T (Y - X\beta) \quad (3)$$

By minimizing the sum of squares of the errors in equation (4), the OLS estimator is produced, which has the following form:

$$\hat{\beta} = \left(X^T X\right)^{-1} X^T Y \quad (4)$$

In many practical investigations, a linear regression model often includes a large number of explanatory variables. Despite its widespread use, the OLS approach has limitations in OLS estimations.[2]

Outliers in Regression Analysis

In regression analysis, outliers can have a big effect on the dependent variable, which can distort the findings and cause misunderstandings. There are several approaches, each with advantages and disadvantages, for identifying and controlling these outliers. "Outliers increase residual variability and induce heteroscedasticity", which distorts parameter estimates. Outliers in the dependent variable can have a substantial impact on regression analysis. By reducing the influence of outliers on the regression model, the Weighted Least Squares approach successfully resolves this problem and generates more accurate estimations.[5]

This huge residual is taken into account and has a significant impact on the average value of the squared residuals, which is minimized using the least squares approach.[2]

Usually, two distribution types are employed to produce contaminated data. The second type, known as the contaminating distribution $f_{c(.)}$ and (g), is the ratio of contamination by distribution $f_{c(.)}$, followed by the distribution of an arbitrary observation, the first form, known as the primary distribution $f_{dc(.)}$, produces clear data.

$$\mathrm{f}_{\mathrm{mix}}(.) = (1 - g) \times f_{dc(.)} + g \times f_{c(.)} \quad (5)$$

Robust Estimation in Regression

Robust regression is the regression technique used when some outliers affect the model or when the residual distribution is abnormal. This method is essential for evaluating data affected by outliers to build robust models. Researchers looked at the widely held belief that regression models don't meet the regression assumptions, and it seemed that the transformation wouldn't eliminate or mitigate the effect of outliers, ultimately producing forecasts that were skewed. Robust regression is the best method in these circumstances since it is unaffected by outliers.

Robust regression is used to find outliers and generate results that are resistant to them. The effectiveness of a linear regression model can be improved in a number of ways when the data are tainted. The concept of a robust estimator, which is resistant to excessive influences from anomalous data points, is often used in these techniques. Numerous methods for bolstering estimates in the face of harsh observations are provided by the extensive body of literature in this area.[1]

One trustworthy method for estimating regression is M-estimation; the letter M in M -estimation stands for the estimate of the maximum likelihood type. M-estimation, first presented by Huber in 1964, is the most often used generic technique for robust regression and is almost as effective as OLS.

$$y_i = \alpha + \beta_1 x_{i1} + \beta_2 x_{i2} + \cdots + \beta_k x_{ik} + \varepsilon_i$$

$$x_i' \beta + \varepsilon_i \quad (6)$$

for the i-th of n observations. Given an estimator b for β, the fitted model is

$$\hat{y}_i = a + b_1 x_{i1} + b_2 x_{i2} + \ldots + b_k x_{ik} = x_i' b \quad (7)$$

y_i: is the actual observed value.
$\hat{y}_i$: is the fitted value, obtained by substituting the predictor values into the estimated model coefficients $b_1, b_2, \ldots, b_k$
x_i: is the vector of predictor variables for observation i.
b: is the vector of estimated coefficients.

and the residuals are given by

$$e_i = y_i - \hat{y}_i \quad (8)$$

With M-estimation, the estimate b is determined by minimizing a particular objective function over all b,

$$\sum_{i=1}^{n} \rho(e_i) = \sum_{i=1}^{n} \rho\left(y_i - x_i' b\right) \quad (9)$$

Each residual's contribution to the goal function is given by the function ρ. The following characteristics should be present in a decent ρ.

- Always non-negative $\rho(e) \geq 0$
- Equal to zero when its argument is zero, $\rho(0) = 0$

- Symmetric, $\rho(e) = \rho(-e)$
- Monotone in $\rho(e_i) \geq \rho\left(e_i^{'}\right) for \left|e_i\right| > \left|e_i^{'}\right|$ (Fox, J. and Weisberg, S., 2002)

Objective function in M-estimation

The regression coefficients β are estimated by solving:

$$\min_{\beta} \sum_{i=1}^{n} \rho(e_i) \tag{10}$$

Where $\rho(e_i)$ is a loss function, and e_i is the residual for observation i:

$$e_i = y_i - X_i'\beta \tag{11}$$

The derivative of ρ with respect to residuals, denoted as $\psi(e_i)$, determines the influence of each observation:

$$\psi(e_i) = \frac{\partial \rho(e_i)}{\partial(e_i)} \tag{12}$$

$\psi(e_i)$ is also known as the influence function, which controls how much an observation contributes to the solution.

Weighted Function (WF)

Regression models use weighted functions to assign varying levels of significance to data points, which influences how well the model fits the data. This is especially useful when some observations are more reliable or relevant than others. The following is a list of the common weighted function types used in regression. In general, a mathematical or computational function that assigns a higher weight to some inputs or components is called a weighted function. Each component's impact on the outcome is changed by these weights. Statistics, machine learning, economics, and other optimization issues frequently use weighted functions.[6]

This modification makes a more precise parameter estimate possible, especially in datasets where outliers have the potential to distort results.[7,8]

Andrew's Weighting Function

After a critical value πc, Andrews' weight function grants zero weight to residuals and smoothly diminishes the weight of outliers. Because of this, it works especially well for handling data that has large, scattered outliers.

The Andrews weighting function, which has the following definition, should be utilized as the weighting function:

$$w(e_i) = \begin{cases} \dfrac{sin\left(\dfrac{e_i}{c}\right)}{\dfrac{e_i}{c}} & |e_i| \leq \pi c \\ 0 & |e_i| > \pi c \end{cases} \tag{13}$$

The value of $c = 1{,}339$ is the tuning constant. Iteratively reweighted least squares can be used in an iterative procedure to get parameter estimates in the M-estimator approach.[9]

WT

WTs are used in wavelet processes to analyze signals, time series, and regression data. A signal can be broken down into several frequency components and time scales using mathematical tools called WTs.

Programming languages or tools that facilitate wavelet analysis are frequently used when implementing a wavelet process.[10,11]

Symlets wavelet

Symlets are orthogonal wavelets proposed by researcher Daubechies, who introduced modifications to the Daubechies family to enhance symmetry while maintaining wavelet simplicity. These wavelets are symmetric and retain the same properties as Daubechies wavelets, with wavelet function orders ranging from 2 to 8, as shown in Figure 1.

Discrete WT (DWT)

A popular observation processing tool, the discrete wavelet transform (DWT) finds utility in various fields, including computer science, mathematics, engineering, and science. DWT provides a multiresolution representation of an observation by breaking it down using scaled and shifted copies of a compactly supported basis function (mother wavelet). The DWT of y, as determined by the formula (14), is given as a vector of observations y, where k is an integer.

$$D = dy \tag{14}$$

D is a vector with (n*1) dimensions that includes both scaling and wavelet coefficients, where w is a wavelet matrix with (n*n) dimensions. It is possible to structure the wavelet coefficient vector into (k + 1) vectors.[12]

$D = [d1, d_2,\ldots,d_k, V_{k0}]^T$. The details are appended with the details of the most recent decomposition at each DWT after the approximation coefficients are separated into bands using the same wavelet as previously, as shown in the following formula:

$$Y = Dd^T = \sum_{k=1}^{k_0} D_k^T D_k + V_{k_0}^T V_{k_0} \tag{15}$$

The inverse DWT can recreate the observations from the denoised data (lowering the contamination) at each level (k).[13]

Thresholding method

In signal and image processing, thresholding is a technique that sets values above or below a threshold to particular levels to simplify or segment data. For instance, sure thresholding successfully eliminates noise and preserves pertinent information in the context of wavelet shrinkage by setting coefficients below a pre-determined threshold to zero.[14]

Thresholding rules

In information and image processing, particularly in wavelet shrinkage, a number of thresholding methods are frequently employed. Two common types are:[15]

1. Hard thresholding
2. Soft thresholding

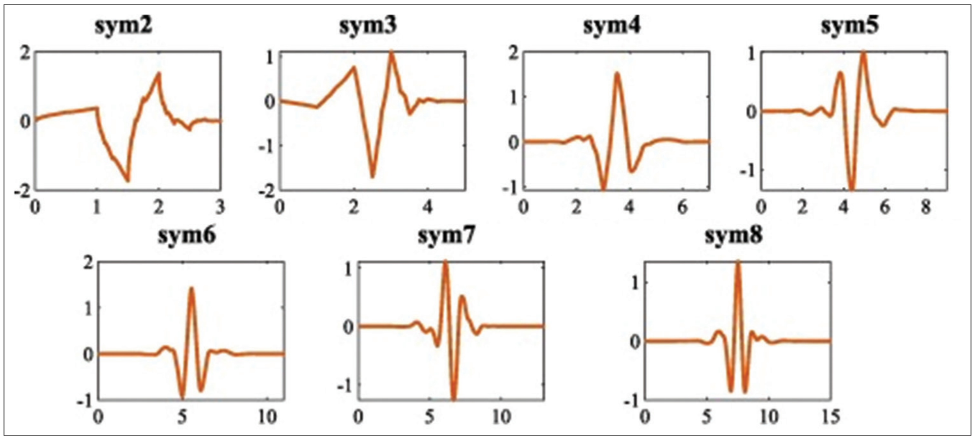

Figure 1: Symlets for eight orders in the wavelet functions.[8]

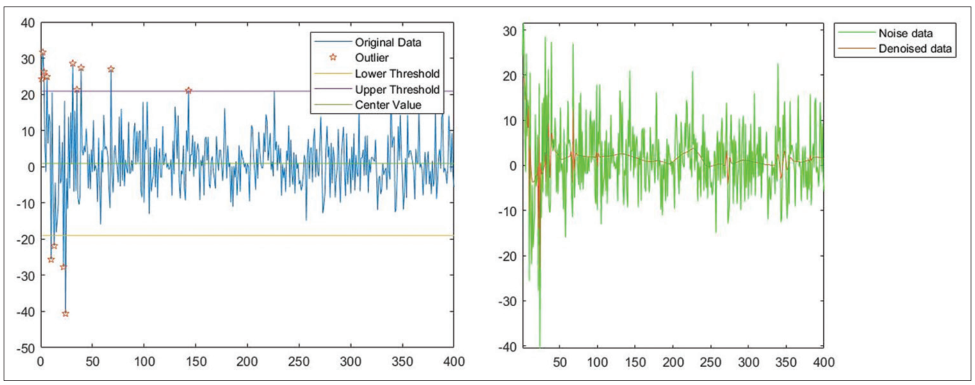

Figure 2: The outliers plot and wavelet smooth plot where (5%) outliers, k = 8, and σ = 1.2 for (n = 300)

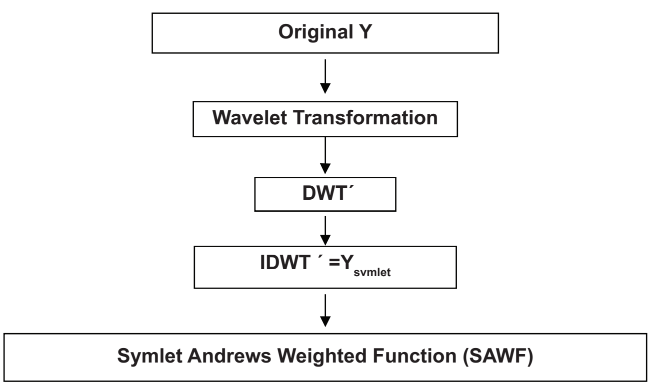

Diagram 1: Proposed an algorithm

The properties of the data and the intended balance between noise reduction and the preservation of significant features determine which thresholding rule is best.[16]

Soft thresholding

Soft thresholding is a method used in signal and image processing, especially for wavelet shrinkage. It entails decreasing the wavelet coefficients"magnitude by a certain amount without precisely setting them to zero. The definition of the soft thresholding function is as follows.[17,18]

$$D_n^{(s)} = sign\{D_n\}\left(|D_n| - \vartheta\right) \qquad (16)$$

Process of Hybrid Symlet wavelet with Andrews Weighted Function

Researchers proposed a hybrid WT with the Andrews weighted function in the robust, to deal with the contamination problem (outliers and noise) for the multiple linear regression model, which depends on the wavelets after treating them with a threshold. And then, using the outputs to find the inverse of the DWT, get filtered data, and later use this data, modified to estimate parameters by the Robust method of multiple linear regression models. Furthermore, root mean square error (RMSE) can be calculated and compared with the aforementioned classical methods.[19,20]

To remove outliers or noise from the values of observations of the response variable, one of the types of thresholds, such as (hard or soft), is usually used by shrinkage of the detail coefficients, which can be obtained by recovering the original observations and splitting them into two components using wavelets. The first represents the sum of the coefficient details, while the second represents the smoothing coefficients based on multiple resolution analysis,[21,22]

$$Y = d\acute{D} = \sum_{j=1}^{j_0} d_j D_j + v_{j0} \acute{V}_{J0} \qquad (17)$$

One of the well-known methods estimates the threshold level, including the fixed threshold method at level j = 1 only

Table 1: ARMSE values for Robust M-estimation using the Andrews weighted function and Symlet Andrews weighted function when (σ=1.2, k=3)

n	Weighted function robust M-estimation	ARMSE %5 outliers	ARMSE %15 outliers
n=100	Andrews weighted function	2.8634	3.4289
	Symlet Andrews weighted function	2.5824	3.2029
n=200	Andrews weighted functions	2.4818	2.8414
	Symlet andrews weighted function	2.0928	2.5283
n=300	Andrews weighted functions	2.1891	2.3728
	Symlet Andrews weighted function	1.6503	1.9589

ARMSE: Average root mean square error

Table 2: ARMSE values for Robust m-estimation using the Andrews weighted function and Andrews Symlet weighted function when (σ=3.5, k=3)

n	Weighted function Robust M-estimation	ARMSE %5 outliers	ARMSE %15 outliers
n=100	Andrews weighted function	5.0065	6.9568
	Symlet Andrews weighted function	4.3454	5.7777
n=200	Andrews weighted functions	4.9712	5.3466
	Symlet Andrews weighted function	3.2345	4.5701
n=300	Andrews weighted functions	4.3746	4.2678
	Symlet Andrews weighted function	2.2341	3.4684

ARMSE: Average root mean square error

Table 3: ARMSE values for Robust M-estimation using the Andrews weighted function and Symlet Andrews Weighted function when (σ=1.2, k=8)

n	Weighted function Robust M-estimation	ARMSE %5 outliers	ARMSE %15 outliers
n=100	Andrews weighted function	3.6146	4.3445
	Symlet Andrews weighted function	3.2311	4.2492
n=200	Andrews weighted functions	3.0794	3.3489
	Symlet Andrews weighted function	2.8777	3.4621
n=300	Andrews weighted functions	2.4456	3.0680
	Symlet Andrews weighted function	2.2055	2.7432

ARMSE: Average root mean square error

(D_1). The DWT coefficients of the modified wavelet, typically represented by $(\widetilde{D})$, can then be obtained by applying the soft threshold to the DWT coefficients and returning the remaining

Table 4: ARMSE values for Robust M-estimation using the Andrews weighted function and Symlet Andrews weighted function (SAWF) when (σ=3.5, k=8)

n	Weighted function Robust M-Estimation	ARMSE %5 outliers	ARMSE %15 outliers
n=100	Andrews weighted function	7.3006	9.0979
	Symlet Andrews weighted function	6.0098	7.5678
n=200	Andrews weighted functions	6.1759	7.6213
	Symlet Andrews weighted function	4.7844	6.3422
n=300	Andrews weighted functions	5.4567	6.1030
	Symlet Andrews weighted function	3.7789	4.1288

coefficients to the vector elements (D). This allows for the recovery of the treating response variable's observations.

$$Y_{symlet} = d'\widetilde{D} \quad (18)$$

Depending on the wavelet matrix, such as (coif 2), the values of (observations of the processed dependent variable) can be obtained, and a weighted function based on wavelet analysis:

SAWF

$$w\left(e_{(AS)i}\right) = \begin{cases} \dfrac{sin\left(\dfrac{e_{(AS)i}}{c}\right)}{\dfrac{e_{(AS)i}}{c}} & \left|e_{(AS)i}\right| \le \pi c \\ 0 & \left|e_{(AS)i}\right| > \pi c \end{cases} \quad (19)$$

which are used with the independent variables in estimating the parameters of the multiple linear regression model, depending on the methods, i.e.:

$$\underline{\hat{\beta}}_{robust(AS)} = (X'WX)^{-1}X'WY_{symlet} \quad (20)$$

The steps of the Symlet wavelet with Andrews' weighted function are summarized in the following algorithm, which also modifies the dependent variable (y) to a new dependent variable based on the wavelet, as shown in Diagram 1:

RMSE Criterion

Regression models created with a particular data sample can be compared using their mean square error (MSE). A lower MSE indicates a higher quality regression model. The following is the name given to the MSE:

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n}\left(y_i - \widehat{y}_i\right)^2}{n}} \quad (21)$$

Y_i: is the actual value for the i_{th} observation.
Y_i: is the predicted value for the i_{th} observation.
N: is the total number of observations. (Sulaiman, N.G. and Rahim, A.G., 2022.)

APLICATION PART

This chapter presents a comparison between the wavelet-based improved weighted function and the classical weighted function. Since robust regression largely depends on the selection of the weight function, this analysis employs one of the robust techniques, namely, robust estimation. To lessen contamination in our data, we then merged two functions. The weight function is essential to robust estimates for estimating parameters in multiple linear regression models, especially when Outlier is present.

To ascertain which weight function is more effective for parameter estimation in robust regression, the study includes an experimental phase that uses simulations. We use the comparison metric of average RMSE (ARMSE) to evaluate the effectiveness and precision of the estimated models. Statistical software tools, such as MATLAB R2021b, which is used for simulation, are used to generate and analyze the results as shown in Tables 1-4.

Simulation

The combination under discussion is repeated (1000) times for various scenarios using simulation of the trials as follows:

1. Three sample sizes (100, 200, 300)
2. Standard deviation of the noise (1.2, 3.5).
3. The number of explanatory variables, denoted by k, can be (3 or 8).
4. The outliers at these rates (5% and 15%). Using the following formula, as shown in Figure 2 :

$$fmix\,(.) = (1 - g) \times f_{dc}(.) + g \times f_c(.) \tag{22}$$

This table displays the ARMSE when (σ = 1.2, k = 3) with Outlier levels for sample sizes (n = 100, 200, 300) and simulated data (5%, 15%). The most practical approach is to compare the Andrews Weighted function in the robust estimate with the Hybrid SAWF. The smaller ARMSE is an improved measure for comparing models to estimate parameters.

This table displays the ARMSE when (σ = 3.5, k = 3) with Outlier levels for sample sizes (n = 100, 200, 300) and simulated data (5%, 15%). The most practical approach is to compare the Andrews weighted function in the robust estimate with the Hybrid SAWF. The smaller ARMSE is an improved measure for comparing models to estimate parameters.

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This table displays the ARMSE when (σ = 3.5, k = 8) with Outlier levels for sample sizes (n = 100, 200, 300) and simulated data (5%, 15%) Outliers. The most practical approach is to compare the Andrews weighted function in the robust estimate with the SAWF. The smaller ARMSE is an improved measure for comparing models to estimate parameters.

CONCLUSION

Based on the outcomes of the practical portion and the simulation data, the following conclusions have been reached: The SAWF is the most suitable method for all instances included in this study when compared to the Andrews weighted function; the suggested approach addresses the outlier issue; however, as the sample size is expanded, the ARMSE value changes, and although ARMSE results remain lower than those obtained using the conventional technique, they marginally increase at a 15% outlier level after decreasing at a 5% outlier level.

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