Research Article

Applying the Bayesian Technique in Designing a Single Sampling Plan

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ABSTRACT

The Bayesian sampling plans for production inspection are considered a technique of sampling inspection techniques for determining the characteristics of the sampling plan based on the assumption that the rate of defectives is a random variable that varies from one production batch to the next, resulting in a probability distribution \( f(p) \) that could be determined based on experience and the available quality information available. As part of this study, the parameters of a single Bayesian sampling plan \((n,c)\) were derived using the beta-binomial distribution and compared with those of other single sampling plans. Researchers have identified (ALA company for soft drinks), which handles product quality control. One hundred and twenty production batches were selected, and the size of the batch and the number of defective items were used to determine the proportion of defective items, given that the variable varies randomly from one production batch to the next. Bayesian and decision-making models can be implemented to create a single sampling inspection process that is close to the actual quality level. The researchers discovered that when the decision-making model was used, the sample size was minimal compared to other inspection plans, leading to a low inspection cost.

Keywords: Statistical quality control, average sample number, acceptance quality level, operating characteristic, Bayesian sampling plans

INTRODUCTION

Among the statistical tools used to control and monitor the quality of production are control charts and sampling inspection plans. Regarding sample testing plans, they are an accurate and appropriate method to obtain an estimate of the presence of one or more characteristics among the produced units. This is achieved by examining a small percentage of the production, which is randomly selected to determine whether to accept or reject production based on the results of the sample drawn. There are two methods of examination using the sampling method: The discriminatory examination method (by attributes), where the produced units are classified into defective and good units, or by the variable examination method (by variables) based on a standard such as height or weight.

The examination may be based on the researcher's or examiner's experience, in addition to any other available information about the production process (prior information). This approach leads to the use of Bayesian theory and decision theory, and the adoption of Bayesian estimation for the quality parameter of production, depending on the loss function, to avoid making wrong decisions regarding the production process.¹,²

Due to the effort, cost, and time required to conduct a comprehensive examination, as well as cases known to statisticians where the above examination can be applied, the sampling examination method was used to evaluate the performance of the quality control department in the company. Since most of the staff working in this department are not statistical experts and are not familiar with statistical methods in the field of quality control, they rely on the accumulated experience of their members, examiners, and laboratory workers.

For the aforementioned reasons, this research aims to shed light on the methods of sample examination and their use in estimating sample size and designing sampling examination plans that can minimize spoilage and reduce costs by making the right decision to accept or reject the produced batch.

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Copyright © 2023 Dler H. Kadir, AbdulRahim K. Rahi. This is an open-access article distributed under the Creative Commons Attribution License (CC BY-NC-ND 4.0).
This research aims to investigate the method of sampling and decision-making regarding the acceptance or rejection of batches. This will be achieved through the design of a discriminatory sampling examination plan using the Bayesian method and applying it to the production of Pepsi Ala Soft Drinks Company to make informed decisions on whether to accept or reject the produced batch.[9,4]

**BAYESIAN SAMPLING PLANS**

One of the methods of sample examination is the Bayesian sampling plan, which is an alternative to the comprehensive (classic) examination, as well as the single, double, and sequential sampling plans. However, Bayesian sampling plans differ from single, double, and sequential sampling plans in that they deal with specifying plan parameters based on the percentage of defects in the product with random variation. This percentage changes from one production batch to another, as it has a probability distribution that can be determined from previous experience and available information on quality.[1,5]

Bayesian plans have emphasized the importance of utilizing available information on quality, and this information is referred to as prior distributions. The prior distributions are used to obtain the post-quality posterior distribution, assuming that the type of sampling distribution being studied may be binomial with parameters \((n,p)\), poisson with parameter \((\lambda)\), or normal with parameters \((\mu,\sigma)\). Bayesian plans are of great importance in examining products, and it is necessary to provide a brief explanation of this topic to reach a Bayesian sampling plan \((n, c)\) that reduces the total cost function of quality control. The cost function is the sum of inspection costs, costs of rejecting non-defective units (good), and costs of accepting defective units (bad).[6]

**Bayesian Plans Concept for Product Inspection**

Bayes’ plans have been named by this name in relation to the scientist (Thomas Bayes) (1702–1761), where Bayes is the first to use the prior distribution for defective percentages in statistical inference, and since 1960, attention began to focus on Bayes’ plans to test the product.

Bayesian plans are named after the scientist Thomas Bayes (1702–1761), who was the first to use prior distributions for defective percentages in statistical inference. Since the 1960s, attention has been focused on Bayesian plans for testing products. In 1964, the scientist Hald (1981) was able to develop a model for the total cost function of quality control. Through this model, the parameters of the sampling plans are determined when the quality is fixed or the random variable has a prior distribution (prior distribution).[7,8]

In 1968, the scientist Hald presented a model for Bayesian plans to examine products. This model is used to obtain the parameters of the individual Bayesian plan \((n, c)\) by reducing the standard cost function of quality control, which is:

\[
R(N, n, c) = n(p_1 - p_m) + (N - n)
\left[ \int_a^b (p_1 - p) Q(p) w(p) dp \right]
\]

Where \((R = P + Q)\), denotes sample costs.
\(P_m\): The standard cost in cases of rejection and acceptance.
\(P_r\): The average cost of examination per unit in both cases of rejection and acceptance, and that:

\[
P_r \geq P_r \geq P_m, \quad 0 < P_r < 1, \quad 0 < P_m < 1
\]

On equation (1) If the value of \((P)\) is very small, we use the poisson distribution, that is:

\[
b(x, np) = \frac{e^{np}(np)^x}{x!}
\]

And we change

\[
z = \frac{D}{P}, \quad m = np, \quad M = NP
\]

The function of the form will be:

\[
B(c, np) = B(c, mz) = P(z)
\]

**Total Cost Function for Quality Control**

The scientist Hald (1981) developed a model that included the total costs of examination, acceptance, and rejection. The model aims to determine the individual sampling plan \((n, c)\) for examining batch \(N\) of the product of quality \(p\) by applying filtering examination. He expressed these costs in the following formula:

\[
h(x, X, p, N, n, c) = nS_1 + nS_2 + (N-n)A_1 + (X-x)A_2, \quad x \leq c (4)
\]

\[
h(x, X, p, N, n, c) = nS_1 + nS_2 + (N-n)R_1 + (X-x)R_2, \quad x > c (5)
\]

The examination and repair costs, in addition to the costs resulting from accepting the quantity \((N-n)\) remaining after drawing the sample, represent the first part. The examination and repair costs, in addition to the costs resulting from rejecting the quantity \((N-n)\), represent the second part. Equating equation (4) with equation (5) results in:

\[
P_r = \frac{X - x}{N - x} = \frac{R_1 - A_1}{A_2 - R_2}
\]

The average cost in both cases of acceptance and rejection is equal to:

In the event of acceptance

\[
n(S_1 + S_2p) + (N-n)(A_1 + A_2p)
\]

in case of refusal

\[
n(S_1 + S_2p) + (N-n)(R_1 + R_2p)
\]

And by entering the probability of acceptance of the product, \(P(p)\) and the probability of rejection, \(Q(p)\) since:

\[
P(p) = P(x \leq c) = \sum_{x=0}^{c} b(x, n, p) = B(c, n, p)
\]

\[
Q(p) = 1 - P(p) = P(x > c) = \sum_{x=c+1}^{n} b(x, n, p) = E(c + 1, n, p)
\]

Thus, the cost rate \((p)\) is obtained in the cases of acceptance and rejection as follows:
The beta distribution is considered one of the important statistical distributions in Bayesian plans. In this distribution, the percentage of defects is a random variable with a beta distribution, and its parameters can be estimated using the moment method. The probability function of the random variable \( p \) with \((\alpha, \beta)\) parameters takes the following form:

\[
f(p, \alpha, \beta) = \frac{1}{B(\alpha, \beta)} p^{\alpha-1}(1-p)^{\beta-1} \quad 0 < p < 1
\]

\[
= 0 \quad OW
\]

In many cases, it cannot be assumed that \( p \) remains constant from one batch to another. For example, consider a machine used to produce a specific unit. After producing a batch, the machine is checked and put back into operation. For each batch, \( p \) can be determined according to certain distributions, which are referred to as the initial distributions of \( p \). For this reason, the beta distribution was chosen as the initial distribution of \( p \) with \((\alpha, \beta)\) parameters. The expected value of the random variable for the beta distribution is \( \alpha/ (\alpha+\beta) \). This means that \( \beta \) must be greater than \( \alpha \), and when estimating \((\alpha, \beta)\), it must be rounded to the nearest integer. If the estimated value lies between 0 and 1; then, it must be rounded to the expected value of \( p \), to the nearest integer. This method allows us to use standard tables to determine the sampling plan and also shows us why the plans are not clearly sensitive to small changes in the parameters without taking into account the appropriate initial distributions.

### Direct Formulas for Determining Individual Base Plan Parameters by Decision Model

To define the parameters of the single Bayesian Economic Statistical (BES) plan \((n, c)\) for product inspection, a formula for the expected risk must be developed and specified directly for the parameters of the sampling plan. We can rely on the definition of risk provided by Guthrie and Johns, who defined it as the sum of examination costs plus the loss resulting from wrong decisions. It is well-known that examination costs depend on the volume. It represents the loss resulting from accepting defective units and the loss resulting from rejecting good units. The expected risk formula, under the conditions of binomial sampling, takes into account the distributions of continuous defective percentages, which are as follows:

\[
R[f(p), n, c] = n[(S_2 - R_2)\bar{P} + (S_1 - R_1) + (A_2 - R_2)] \
+ \int_0^{\beta} (P - P)Q(P)f(p)dp + \
N(R_1\bar{P} + R_1 + (A_2 - R_2)] \int_0^{\beta} (P - P) f(P) dp \
+ (N - n)\left\{\frac{1}{2n}(R_1 - A_1)\right\}(1 - p_1) f(p_1)
\]

When neglecting the upper limits, equation (8) is reduced to the following form:
\[ R[f(p), n, c] = An + BN + C(N - n) \frac{1}{n} \]  

(9)

So:

\[ A = [(S_2 - R_2) \bar{P} + (S_1 - R_1) + (R_2 - A_2)] \int_0^p (P - P_f) f(P) dP \]

\[ B = R_2 \bar{P} + R_1 + (A_2 - R_2) \int_0^p (P - P_f) f(P) dP \]

\[ C = 1 \frac{1}{2} (R_1 - A_1) (1 - P_f) f(p = p_r) \]

\[
\int_0^p (P - P_f) f(P) dP = \tilde{IB}_n (\alpha + 1, \beta) - P, IB_n(\alpha, \beta)
\]

When we derive equation (9) with respect to \( n \) and equate the derivative to zero, we obtain the optimal value of \( n \) (n*). To find the sample size necessary to examine batch \( N \) of the product, assuming binomial sampling and that the defective percentages change from one production batch to another, we can use the following formula: The random variable has a prior distribution equal to beta with \((\alpha, \beta)\) parameters, and the optimal value of \( n \) can be obtained from the following equation.

\[
n^* = \left[ \frac{2B(\alpha, \beta)(S_2 - R_2) \bar{P} + (S_1 - R_1) + (R_2 - A_2)}{(\tilde{IB}_n (\alpha + 1, \beta) - P, IB_n(\alpha, \beta))} \right]^{1/2}
\]

When the values of \((\alpha, \beta)\) are integers, the relationship between the incomplete beta function and the cumulative binomial function can be derived. This relationship can be used to extract the following:

\[
\tilde{IB}_n (\alpha, \beta) = \sum_{x=\alpha}^{\alpha+\beta-1} C_{\alpha+\beta-1}^x P_r^x (1 - P_r)^{\alpha+\beta-x-1}
\]

\[
\tilde{IB}_n (\alpha + 1, \beta) = \sum_{x=\alpha+1}^{\alpha+\beta} C_{\alpha+\beta}^x P_r^x (1 - P_r)^{\alpha+\beta-x}
\]

The number of acceptance can be obtained from the following relationship:

\[
c = n\bar{P}_r - \frac{2}{3}
\]

\( \bar{P}_r \): Critical quality level.

**Direct Formulas for Determining Single Bayes Plan Parameters from Hald Model**

Determining the parameters of individual Bayesian plans \((n, c)\) for product inspection requires lengthy iterative calculations to find the values \((n, c)\) that achieve the smallest expected total cost or the smallest standard cost. Research in this area has focused on finding formulas that efficiently and quickly obtain optimal parameter values. Continuous studies and research have led to formulas used directly for large production batches where the quality of the batches is a random variable with a prior distribution \(f(p)\), which is continuous and differentiable at points adjacent to a point \((p = p_r)\). The discontinuous distribution of defective percentages was discussed by the scientist Hald in 1965, and direct formulas were developed by Hald in 1968, which were supported by auxiliary tables. These formulas can be used to extract Bayesian plans in distributions such as gamma-poisson and beta-binomial.

\[ K_m = \int_0^\infty (A_1 + A_2 P) f(p) dp + \int_0^\infty (R_1 + R_2 P) f(p) dp \]

\[ K_m = \int_0^\infty (A_1 + A_2 P) f(p) dp + \int_0^\infty (R_1 + R_2 P) f(p) dp - \int_0^\infty (R_1 + R_2 P) f(p) dp \]

\[ K_m = K_r - (A_1 + R_2) d_m \]

Among them, we find that:

\[ d_s = \int_0^{\bar{P}} (P - P_f) f(p) dp \]

The value of \(d_s\) in terms of \(d_s\), as:

\[ d_s = d_s - P, + \bar{P} \]

\[ d_s = d_s + (S_1 - R_1) \bar{P} + (S_2 - R_2) \bar{P} + (A_2 - R_2) \]

\[ K_m = K_r - (A_2 + R_2) d_s \]

So it is:

\[ NK_m = K_r - (A_2 + R_2) d_s \]  

(11)
\[ R(N,n,c) = \frac{K(N,n,c) - K_n}{A - R_2} \] (12)
\[ R(N,n,c) = nds + (N - n)d(n) \]
\[ d(n) = d_1 + \int_0^1 (P - P_1) B(c,n,p) f(p) dp \]

Depending on the equation
\[ \lambda_1 = \frac{p^* q^*(A_2 - R_2)}{2B(\alpha, \beta)(k - k_n)} \]
\[ \lambda_2 = \frac{3(\alpha, \beta)^2 - 11(\alpha, \beta) - 2 - 3(\alpha - 1)}{p, q, r - 1} \]

The value of \( n \) necessary to check the batch \( (N) \) is the value defined by the following relation:
\[ n^* = \lambda_2 \sqrt{N} + \lambda_2 \]

As for the number of acceptance of \( c \), it is extracted from the following relationship:
\[ c^* = n^* p_1 + \beta_1 \]

Whereas:
\[ \beta_1 = \hat{\beta} p_1 - \hat{\alpha} q_1 - \frac{1}{2} \]

To find the value of the standard cost function \( R_0(N) \) corresponding to the optimal sampling plan \((n^*, c^*)\), which we will symbolize \( R_0(N) \), the following equation will be relied on:
\[ R_0(N) = (2n^* - \lambda_1^2 - \lambda_2) ds \]
\[ d_1 = \frac{k_1 - k_n}{A_2 - R_2} \]

Therefore, the value \( K(P) \) of the expected total cost of quality control is equal to:
\[ k(P) = R_0(N)(A_2 - R_2) + NK_n \]

**RESULTS**

The process of improving product quality requires relying on modern scientific methods. By utilizing modern practical methods and adhering to standard specifications, products can be made suitable and conform to the desired specifications of consumers. This not only elevates the status of the production facility in local and global markets but also enhances the value of these products in these markets. Therefore, it is essential to establish quality control requirements fully. This includes prioritizing standard and manufacturing specifications for input, process, and output elements. One of these requirements is to identify and provide a scientific method for examining materials and products to ensure the reduction of damage and to ensure the regular flow and handling of circulation during production processes in the facility.

**Application of the Decision-Making Model**

Based on the decision-making model, a set of Bayesian economic-statistical (BES) plans was developed to test the product, depending on the previous distribution of defective percentages (Beta-Prior). The parameters of the individual BES plan were determined according to the decision-making model to obtain the values of \( n \) and the acceptance number \( c \). The inspection plans for this product are shown in Table 1, taking into account the levels of quality and sizes of production batches.

The parameters of the sampling plan required to check the daily production of the 1.5-L (Pepsi Ala) product of quality \((X = 0.005349)\) and value \((P = 0.00617)\) were extracted using this model. The values obtained were \((n, c) = (1495, 14)\), and the expected risk value for the sampling plan was also determined to be \((1495, 14)\). The value of \( R(f(p), n, c) \) found to be equal to $43,210.

**Hald Model Application**

Since the distribution of the defective percentages \( f(p) \) is one of the continuous and derivable distributions at point \((P = P_1)\), a set of Bayes plans will be extracted from the direct formulas created by (Hald), as the set of Bays plans necessary to check the product, which is defined from equation (14), must be calculated before that each of:

Examination cost rate per unit
1. \( k_1 = \sigma_1 + \sigma_2 \hat{p} = 0.0011 + (0.03)(0.005349) = 0.00126 \)
2. \( k_2 = \sigma_1 + \sigma_2 \hat{p} = k_1 \)

Examination cost rate per unit
3. \( 0 + 0.208(0.005349) = 0.00118 \)

As well as the values of the ingredients: (4, 5, 6, and 7):

(4) \( \int_0^P (p - p_1) f(p) dp = p_1 B_p(\alpha, \beta) - \hat{\beta} B_p(\alpha + 1, \beta) \)

(5) \( k_1 = k_1 + (A_2 - R_2) \int_0^P (p - p_1) f(p) dp \)

(6) \( \lambda_2 = \frac{3(\alpha, \beta)^2 - 11(\alpha, \beta) - 2 - 3(\alpha - 1)}{p, q, r - 1} \)

Whereas:

(7) \( p_2 = 0.0061, \quad A_2 = 0.208, \quad R_2 = 0.03, \quad \hat{\alpha} = 25, \quad \hat{\beta} = 4648 \)

\[ \lambda_2 = \frac{3(\alpha + \beta)^2 - 11(\alpha + \beta) - 2 - 3(\alpha + 1)}{p, q, r - 1} \]
Table 1: Bayesian plans to test the product according to the distribution of beta-prior extracted from the decision-making model

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### Table 2: Bayesian plans to test the product extracted from the (Hald) model

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And I extracted my value \((\lambda_1, \lambda_2)\), so the value of \(n\) necessary to check the batch is \(N = 39388\), which is the value specified by the following relationship:

\[
n' = \lambda_1 \sqrt{N} + \lambda_2
\]

\[
n' = (12.17582) \sqrt{39388} + (55.4084) = 2361 \text{ units}
\]

\[
c' = n' \cdot \frac{p}{R}
\]

As for the acceptance number \(c\), it is extracted from the relationship, so that

\[
\beta_i = \frac{R_i - \Delta q}{\frac{1}{2}}
\]

Thus, the value of the acceptance number corresponding to the sample size \((n = 2361)\) is equal to:

\[
\beta_i = (4648) (0.0061) - (25) (0.9939) - 0.5
\]

\[
\beta_i = 3.0053
\]

Accordingly:

\[
c' = 2361 (0.061) + 3.0053 = 17 \text{ units}
\]

Therefore, the single sampling plan necessary to check the production rate is \(N = 39388\), and extracted from the (Hald) model is \((2361.17)\), and this means that the examination of a random sample of \((2361)\) is invalid. If the number of defective (damaged) units in the sample is equal to \((17)\) champion or less all units are accepted, otherwise, the batch is rejected.

As for the total cost of quality control resulting from the sampling plan \((2361.17)\), it will be extracted based on the smallest standard cost \(R_0(N)\) achieved in the optimal sampling plan \((2361.17)\), as

\[
R_0 = \frac{(2n' - \lambda_1^2 - \lambda_2)}{\Delta p} - \frac{s}{(A_2 - R_2)} ds = 0.00954
\]

\[
R_0 = [2(2361) - 148.2506 + 55.4084](0.00954)
\]

\[
R_0 = 4.4162
\]

Therefore, the value of the expected total cost \(k(p)\) of quality control is equal to:

\[
k(p) = R_0 (A_2 - R_2) + NK
\]

\[
= 4.4162(0.178) + 49388(0.001091)
\]

\[
= 42.9731 \text{ $}
\]

Table 2 includes all the results of Bayesian plans to test the 1.5-l. liquid Pepsi product extracted from the (Hald) model according to the previous distribution (Beta), classified according to quality levels \(x\) = 0.001(0.001)0.005 and batch sizes \(N = 10,000(1000)50,000\).

**Calculating the Value of the Defective Fraction in the Unexamined Quantities**

According to the decision-making models and the (Hald) model, a group of Bayesian plans have been extracted to test the product. After this extraction, it is necessary to determine the expected value of the expected fraction of the defective
fraction in the unexamined quantities ($N-n$) which will be accepted based on the acceptance of the sample, and then, we will depend on the value of the average of the subsequent distribution for the defective lineage ($E(p|x)$) when ($X = c$), that is, ($P_n(x)$) and it has become clear to us that the subsequent distribution ($f(p|x)$) is also a house with features $(x+\beta+n, x+\alpha)$, and therefore, it is

$$E(p | x) = P_n(x) = \frac{x+\alpha}{\alpha+\beta+n}$$

And when $X = c$ is

$$P_c(c) = \frac{c+\alpha}{\alpha+\beta+n}$$

The following Table 3 includes a comparison of BIS plans according to the (Hald) model and the decision-making model and at the level of quality ($X = 0.005349$) and the values of ($Pn(c)$) and for each of the plans of the decision-making model and the (Hald) model.

Table 3 shows that the expected value of the fraction of defective items in the accepted quantities ($N-n$) is 0.6% according to both the decision-making model and the (Hald) model. This value corresponds to the permissible percentage of defective items (LTPD) approved by Pepsi Company (ALA) for soft drinks. The correspondence of the average value of ($Pn(c)$) with the value of (LTPD) indicates the efficiency of the BIS plans, which take into account all available information about the quality when estimating the quality of subsequent production batches. This congruence underscores the importance and efficiency of the BIS plans. Moreover, the actual production quality level ($X = 0.005349$) shows that the sample size for different batch sizes is smaller compared to other sampling inspection plans, which reduces examination costs and total costs.

**CONCLUSION**

1. The appropriate probability distribution to represent the defective percentages of the actual production is the beta-binomial distribution with a rate of ($0.005349$).
2. After applying the Bayes model in designing the sampling inspection plan, it was found that the parameters of this plan are ($n = 1495$) and ($c = 14$), and for the quality level of the actual production ($X = 0.005349$), we find that the sample size for the different batch sizes is small compared to other sampling plans, which means reducing examination costs and therefore the total costs.

**REFERENCES**